be confirmed by the discussions of the late transit of Venus, the accuracy of my chemical estimate of Sun's mass and distance* will be likewise confirmed.

The generalization of Faraday, as corroborated by the measurements of Weber, Kohlrausch, and Clerk Maxwell, is thus extended to include gravitation, as well as electrostatic and electrodynamic action, in the same category of central force with light (VI), by means of an identical limiting *vis viva*. The simple mathematical correlations make the generalization still broader, so as to embrace heat (IV), chemism, cosmical and molecular aggregation, dissociation, rotation and revolution (V–X), and all central forces (I–X).

The Flow of Water Through an Opening in a Pierced Plate.

By Robert Briggs.

(*Read before the American Philosophical Society, November* 3*d*, 1876.)

The consideration of the subject of the *vena contracta*, or section of a vein of water emerging from an orifice under certain conditions, is made a portion of the proceedings of the Philosophical Society of Glasgow, and appears in their volume X, page 145 et seq. Four papers are published, the first of which is an extract from a letter of William Froude, Esq., C. E., F. R. S., to Sir William Thomson, dated Cheston Cross, Torquay, 20th December, 1875. Mr. Froude is quoted† * * * * "One result I have tried came out well :—The discharge through an introverted cylinder [tube] with keen edge. Here, by theory, the section of the jet ought to be exactly half of the aperture. For the conservation of stream line energy obliges the velocity to be that due to the head, while the conservation of momentum requires that the pressure on the aperture (which is here the sole operative pressure setting in the ultimate direction of the velocity generated) is only sufficient to create as much momentum, say, per second as will be resident in the length delivered per second, of a column of discharge, of half the sectional area of the aperture, if its velocity is that due to the head.

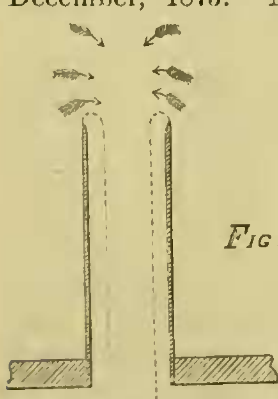


Fig 1

"The cylinder was quite smooth outside and the edge quite keen. The area ratio came out 0.503, 0.502, &c., instead of 0.500, and the little excess was obliterated, if the head was counted, to about one-fourth the diameter below the edge ; as indeed it ought to be (I won't swear to the exact figure

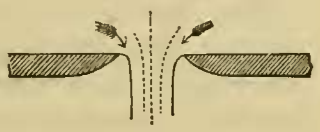
* Proc. S. P. A., xii, xiii, *loc. cit.*

† The entire article is quoted, the hiatus indicated by asterisks exist in the published Proceedings of the Glasgow Society.

one-fourth), because till the motion of the particles is purely parallel to the axis, there must be some accelleration to be effected in the direction of the axis, and this demands the employment of some vertical pressure. * *

"In the *vena contracta* experiment with the thin plates and open air between the plates, the fluid was welcome, if it pleased, to start tangentially to the plane of the aperture as here indicated, and as it appears to do if closely studied. So also with the introverted cylinder; although it was not possible to see what happened I have no doubt that the motion of the particles *next* the edge was vertical upwards, the curvature being only such as the pressure in the contiguous stream would satisfy. If the experiment was not adroitly initiated, the water seized the inner surfaces of the cylinder and run out in an eddied condition, filling the discharge pipe. When, however, it was properly started, the contracted column below issued with beautiful smoothness and symmetry."

Fig 2



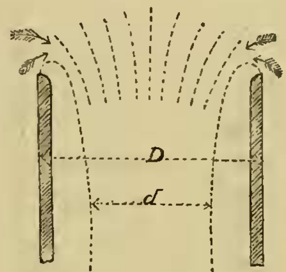


Fig 3

The second paper on the subject is an extract from a letter of Sir Isaac Newton to Professor Cotes, March 24th, 1710–11; from the "*Correspondence of Sir Isaac Newton and Professor Cotes, including Letters from other eminent men,*" published in 1850, by Mr. J. Eddlestone, Fellow of Trinity College, Cambridge, from the originals in the Library of Trinity College. This letter possesses much value as showing Newton's experiment and discussions of the form of the *vena contracta*, and will be found interesting to examine in the proceedings of the Glasgow Society by those who have not ready access to the primary publication. Newton found the *vena contracta* from an aperture in the *side* of a vessel of thin sheet tin plate, five-eighths of an inch in diameter, to have, at the distance of one-half an inch from the hole, a diameter of 21-40ths of an inch. Which was a reduction of diameter of 21-25ths, and of areas of cross section of 0.7058.

A foot note to this letter of Newton by Mr. Eddlestone says, that '*Sectio vena contracta*' was a term used by Jurin, Philosoph. Trans., Sept.—Octo. 1722, p. 185; and that Dan. Bernouilli uses the same term, Hydrodynam., p. 65. Jurin also uses '*vena contracta*'; in all these cases the words denote the reduced section only, while subsequent usage generally applies them to the stream itself as a body, between the orifice and the point of reduced section.

The third paper is a discussion of the *vena contracta* by Professor James Thomson, LL.D., C. E., University of Glasgow, and is a mathematical discussion with six illustrations, intended to demonstrate under certain conditions, as for instance the supposed absence of fluid friction or viscosity, and a supposed great magnitude of vessel and depth of water compared with the dimensions of the orifice, that the jet of water issuing from an orifice in a thin plate, from a conical adjutage either protruding or re-entrant, has a section, where the stream flows out in sensibly parallel lines, of *more* than half the area of the orifice, and that this condition only ceases for a re-entrant nozzle, in the form of a parallel tube as treated of in Mr. Froude's paper. The conclusions of Professer Thomson do not seem to the writer as warranted by the conditions to which he limits his proposition. The value of the assumed force which he denotes by P is by no means satisfactorily exhibited. As the purpose of this paper is to discuss other points, further notice of Professor Thomson's article may be omitted, only it will be assumed that the reader of this refers to the Glasgow Society's proceedings to see for himself what is set forth by the Professor.

The fourth paper is an abstract of remarks by R. D. Napier, Esq., who gave some consideration of this subject, which was published in 1866, in a pamphlet "*On the Velocity of Steam, &c.*," in which he made the general assertion, with some qualifications, that the area of the true theoretical *vena contracta* is half that of the orifice. He says, "I have proved in the pamphlet referred to, that the pressure in the plane of the orifice is nearly half the pressure due to the head, and that from thence to the *vena contracta* [the words are here used in the sense of the section of least area] it gradually diminishes to zero. This diminishing pressure causes increasing velocity, and is thus the direct cause of the *vena contracta*. * * * "About three-eighths of the ultimate velocity and five-eighths of the *vis viva* is imparted to the water outside of the plane of the orifice, and it is absurd to attribute these effects either to what I may call the converging momentum being transferred from one side of the orifice to the other, or to the converging particles preventing the free egress of the stream through the orifice, which are the only views hitherto offered to explain the cause of the *vena contracta*."

This question of the *vena contracta* is a very pretty one in physics, and deserves a more complete and general discussion than it receives in the pages of the Glasgow Society. It should be recognized however, that it does not admit of the simplicity of investigation, either mathematically or experimentally, which the papers of Mr. Froude and Professor Thomson assume. It is impossible to divest the consideration from the fluid friction against the contiguous sides, surfaces, or the edges of the aperture, nor from the fluid friction of the liquid within itself which constitutes *viscosity;* nor yet further, from the frictional resistance to discharge into another medium (the atmosphere in this case); while the absolute strength of water is brought into action in the emerging column to the extent of pressure of the atmosphere. Values for these various elements can be accepted, and the mathematical investigation proceeding from them, would enable a thorough solution of the problem, in place of the extremely partial one essayed in the proceedings of the Glasgow Philo. Soc. Even the effect of dimension of vessel or volume of water with relation to the aperture might be made a part of the investigation and appear in the result.

In such an attempt to find a general solution of the theorem it would at once become evident, that there are two normal forms for the *vena contracta*, viz.: That, when the stream emerges *downwards* from an opening through horizontal edges, and that, where it emerges *upwards* through an opening of the same character. The first of these gives a pencil, whose shape for its longitudinal section at its upper end, or origin, will be controlled by the nature of the aperture, and by the effect of the initial directions of the particles of the effluent liquid (the theoretical *vena contracta, under pressure, but devoid of gravity*); modified by the effect of gravity, which would give to any descending pencil of a fluid, the motion of whose particles shall be established in approximately parallel lines, a hyperloid contour. The second of these will give a sheaf, whose shape at the point of efflux, will be determined by the same laws; while it would now be modified at this point, by the *load* of the emerging fountain, and at the same time the form of the stream above (in this case attaining on some plane an absolutely contracted section), would be that of a hyperboloid sheaf, with both *external* and *internal* lines of definition. If it be supposed in this second instance that the plane of efflux (of the orifice) is slightly deviated from the horizontal, so that the emerging stream is made to take a line out of the perpendicular one, the sheaf form would be disturbed; and at some quite small angle of deviation, a trajectory curve would take its place.

The general course of the stream would then have a modified parabolic curvature—a trajectory curve, which has been frequently discussed—but the exact contour of the pencil is still an open question. It is certain than when passing the point of greatest elevation, it would have, from its retarded motion, its greatest cross-section, and that this cross-section would be a flat oval of peculiar form; and it is probable that beyond this section, on the descending stream, it would become nodal, for the same reasons that a stream emerging from any orifice except a circular one becomes nodal.* In short the complete solution of the problem not only admits and assumes values for all the physical conditions, but it will embrace all directions of efflux from 0° to 180°, where 0° may be taken as the perpendicular direction, either upwards or downwards.

It is possible, for the purpose of illustration, to give some consideration of the *vena contracta* upon hypotheses similar to those of Professor Thomson, and if other conditions are assumed at the same time, an *appreciation* of the phenomenon can be had. In truth the view it is proposed to offer may go further than a mere appreciation, and may be made the basis for support of the other fundamental controlling conditions, and indicate the true line of procedure for mathematical investigation. Let us suppose, with Professor Thomson, that the effect of fluid friction, or viscosity, is neglected; that the magnitude of the vessel and the depth of liquid, is so large in relation to the dimensions of the orifice, that no appreciable velocity is imparted to the mass of liquid by the discharge; that the jet is one issuing downwards (so as to have the cross-section under absolutely uni-

* See article by Weisbach, "Ausfluss," in the "Allgemeine Maschinen Encyclopädie."

form pressure); that the orifice is a circular hole in a thin plate—the flat bottom plate of the vessel—and that the effect of gravity on the stream after emergence be neglected, as well as the atmospheric resistance and the acceleration due to the column of discharge, in the production of a vacuum upon the sectional area; and the following sketch (Fig. 4) gives a general ideal view of the *vena contracta* under these suppositions.

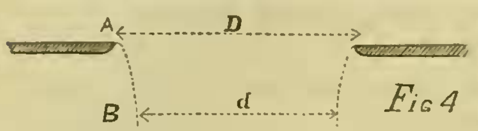


Fig 4

Weisbach* has observed that the actual diameter = d of the vein of emergence from a thin plate, is about 0.8 D; at the point B, which will be found from one-fourth to one-half the diameter (= D) from the plate; and this is accompanied with an efflux, as measured by the quantity of water discharged, of = 0.97v, where v = the velocity of flow = $\sqrt{2gh}$. Now it is an obvious conclusion that at any point on the surface of the *vena contracta* between A, the edges of the plate, and B, the point of minimum section, a particle of water must be in such equilibrium of pressure as to establish its direction of flow, or in other words its curved path; when it becomes apparent that some momentum must have been imparted to such a particle, to induce it to follow in its line of trajectory, instead of following the direction due to gravity, or to the application of the pressure normal to the head, or column of water above it. An attachment to the orifice can be constructed which will exhibit this phenomenon, or rather provide for its occurrence as a matter of necessity, as follows: Let there be an opening in a thin plate as before (Fig. 5), and let this opening be *guarded* or protected by a disc (Z) of the same diameter, = D, let this disc be placed so that its edges (C C) shall be one-eighth the diameter = $\frac{1}{8}$ D (C A) removed from the hole. On these suppositions, if the diameter of the section of least area, = d, be taken at 0.707 D, then the area of the peripheral opening (at C A) will be equal to the area on the plane of (B). The line of effluent stream (A B) may be imagined to be a quadrant of a circle, which will then have of course, a radius, = 0.147 D. Now let the face of the disc Z be made a conoid, so that the areas of the surfaces of the conical frustra a b c d e shall be equal, or in other words, so that all the sections normal to the curve A B shall be equal.

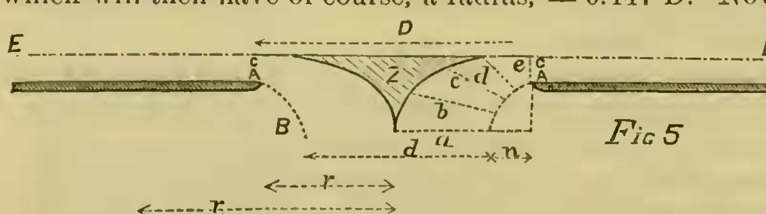


Fig 5

[The co-ordinates for this curve of the face of the conoid Z, in terms of x and y, where x is supposed to have its origin at point of the prolongation of the line CA on the line B; are given by the equation:

$$y = \frac{\sqrt{[r^2 + n^2 \pm \sqrt{(r^2 + n^2)^2 - 4\,n^2 x\,(2r - x)}] - 4(r - x)^2(r + x)^2}}{2(2r - x)}$$

*Weisbach's Mechanics.

In this equation r = the radius of the opening commencing at the origin of x, and n = the radius of the quadrant or corner commencing at the same point.

The assumption of the quadrant of a circle for the path of the effluent particle from A to B, has been made in order to give a simple equation for, and ready comprehension of, the nature of the sections of the stream normal to its face on A B where, by equalities of areas, uniform velocities would subsist; but the real curvature (A B) is obviously parabolic, and the plane of B is infinitely distant from (below) the plane of A. Observation has shown, that at about one-fourth D below the plane of A the least section of *vena contracta* is apparently reached, and that below this plane of section, the pencil of descending current has its sides with only so much divergence from parallelism, as is due, almost entirely, to the acceleration of the falling stream. An elliptical quadrant which shall approximate to the true parabolic curve can be readily substituted by construction (or calculation) for the quadrant of a circle, in the equation above quoted, and the new values for y, will give loci for the curve of the face of the conoid Z to correspond to the substitution. The value of the radius of the minor axis on the line B as determined by observation, may be taken as that of (n) in the equation. By this method a very close approximation towards the true form may be attained.]

It will then result that the efflux from the peripheral opening C A inwards, having any given velocity, will, in every part of the current, until the least section of the *vena contracta* on the plane B is reached have a uniform and constant rate of speed; neither acceleration, nor transformation of head into velocity, will have occurred in the change of direction. If the consideration of the fluid friction, etc., be not taken into the question, and the velocity of efflux at C A is that due to the head, that at B is established and maintained; whence any liquid particle on the surface A B must be in equilibrium of pressure, both from head or momentum in direction of its flow, *in which direction the entire head is transformed into velocity.* The plate, or plane surface of D, may be imagined to extend indefinitely in the directions, E E, in which case the velocity of flow of liquid, interposed between the plate E and the bottom A, will decrease as the radial distances from the edge of the aperture; in inverse ratios of the radius r to any new radii $r_{/}$ $r_{//}$ $r_{///}$: while the height, of liquid column corresponding to the several velocities, = V at r, $V_{/}$, at $r_{/}$, will vary as $\left[\frac{r}{r}\right]^2 : \left[\frac{r}{r_{/}}\right]^2 : \left[\frac{r}{r_{//}}\right]^2$ &c. The pressure or total height is supposed to have been completely transformed into velocity, = V, at the peripheral opening C A, and the stream or sheet of fluid would exert no transverse pressure at C A, either upwards or downwards; while the transverse fluid pressure on the supposed plate or the bottom of the vessel would vary as $\left[\frac{r}{r}\right]^2$ (at C A) to $\left[\left[\frac{r}{r}\right]^2 - \left[\frac{r}{r_n}\right]^2\right]$; r_n being any assumed radial distance from the centre of the opening. Thus if the radius r be taken as one-half inch, and that of r_{11} as five inches (or ten times r), the velocity of flow towards the aperture at r_{10} would be only one tenth of that at r; the head required to produce the velocity at r_{10} would only be one-one-hundredth of that corresponding to the velocity at r; and the pressure of the head remaining on the plate or bottom at r_{10}, would be ninety-nine-one-hundredths of the total head.

These two pressures on the plate and the bottom would be equal and opposite pressures, and if the plate were removed, the unbalanced pressure on the bottom would represent the force P, to which Professor Thomson gives an undefined value. Its total, is of course the sum of the head upon the area of half the opening, and continuing the supposition of removal of the plate, it is encountered and balanced by the *momentum of the descending mass*, so that the bottom would now be in equilibrium of pressure, and the force P, as an unbalanced one, would disappear.

Returning to the examination of the proposition as shown in Fig. 5: the static resistance of the under surface of the conoid Z in a vertical direction against the flow of water in its radial movement towards the centre of the orifice, and while following the path of the under surface of the conoid, is represented in total by the divergence at right angles of the entire effluent stream; = to $\frac{1}{2}$ D of superfice, under the head which has produced the efflux. The reaction of the flow of liquid downwards is also equal to another statical resistance of the same value, and in the same direction; and as the total pressure on the conoid Z from above, is its entire upper surface, under the head of liquid above it; the one pressure above balances the two pressures below, and the conoid itself is in equilibrium.

If it is now assumed that there exists no frictional adhesion of the liquid to the surfaces of the supposed plate, and of the bottom of the vessel, and the vessel is of indefinite extent, so that the velocity of entry at E E is reduced to an inappreciable rate of flow, then the condition of the formation of a perfect *vena contracta* will have been exhibited. The removal of the guide plate E E, and the removal of the bottom of the vessel, and substitution of a re-entrant tube, would replace the supposed frictionless surfaces by liquid mass, which if it is still continued to be supposed devoid of *viscosity*, would enter the peripheral surface C A with the same force, and in the same direction, and would still preserve the same perfect *vena contracta*. The removal of the conoid Z would provide a fluid conoid of the same shape, *or a distribution of internal strains* productive of the same resistance, and (still assuming the perfect liquid) the same perfect *vena contracta* would follow. If however there is admitted to exist a certain adhesion to the bottom of the vessel, or to the surface or edges A A so that the velocity of a particle on A B is less than that fully due to the head; the surface (d) would then become larger than $\frac{1}{2}$ D, the dimension C A would be properly increased to give the corresponding area of efflux, and the conoid Z would also have such a contour as would permit the uniformity of flow of each and every particle of the liquid at unchanged velocity, in any section of the *vena contracta* transverse to the direction of the flow. This increase of dimension of the cross section *d*, and the effect of the descending pencil in accelerating the flow through it, can be taken as sufficient to account for Weisbach's observed value of = 0.8D, and position of least section at = $\frac{1}{4}$ D, as has been before quoted.

It must not be taken for granted that the writer is arguing that the conoid Z actually exists in the water, but it is here assumed for the purpose of showing that all the phenomena of the *vena contracta* are consistent with the supposition. Mr. Froude's "*tangential*" direction for the fluid in Fig. 2, which he says appears if closely studied, is a portion of the proposition, and this discussion exhibits "the imparting of velocity and *vis viva* outside of the plane of the orifice," as alluded to by Mr. Napier

There is one other point worthy of notice in this radial flow of currents towards an orifice, and the radial direction at the edges of the opening. With or without the assumed central, neutral conoid, this flow is in exceedingly unstable equilibrium, especially when in contact with a bottom plate, (the friction or adhesion to which retards the flow), the radial direction may be diverted to a small extent, so that the particle of water where it curves at the point A, or at any other point on the line C A may possess absolute momentum out of the line towards the central axis of the pencil.

The radius of the opening calls for a very slight deviation of entering horizontal current, when its dimension is compared to the area from which this current is derived ; and there is really but the slightest cause for the currents to direct themselves to the exact centre of the orifice. In point of fact the permanency of the *vena contracta* of downward discharge is derived in great degree from the pressure of the atmosphere, which is brought into action, by the descending pencil below it. The effect of a tangential afflux at the peripheral circle C A is to give a rotation to the pencil, which at once accelerates, to some limit of discharge, and obliterates the *vena contracta* The motion of the particles will yet remain limited *in any direction* by the head, but as the stream emerges with a rotary motion, the path of any particle becomes a spiral one, and the whole pencil advances, or is discharged, at a slower rate than is due to the particle velocity. In the case of the re-entrant tube, where the pencil is divested from the effect of gravity, by exhaustion of the air by the effluent stream passing from the tube, it is very difficult to get a *vena contracta*, as Mr. Froude testifies. This action of the tangential afflux is not confined to the emerging stream, but shows itself in the vessel as well, where a whirl is established which involves the entire mass of water enclosed. In the case where the bottom of the vessel has a funnel shape, this whirl sets up with great vehemence, and the centrifugal force of the established current may be sufficient, under favorable circumstances of form, head and dimension of vessel, to displace the entire central portion of the liquid, and the pencil of emergence will become a tube, whose core will be filled by an induced current of air. These phenomena of efflux are only noticed to embrace in my remarks some of the influences which effect the *vena contracta* where the conditions of formation are varied by adjutages, and to make it evident that neither the study of phenomenal nor the mathematical considerations involved, have been exhausted in the papers of the Glasgow Philosophical Society, while the present article does not pretend to do more than to indicate the direction in which enquiry should be pursued.

Remarks upon the Tónkawa Language.

By Alb. S. Gatschet.

(*Read before the American Philosophical Society, Nov.* 17, 1876.)

A small body of Texas Indians, the wretched remains of a once powerful tribe, bears the name of *Tónkawas* or *Tónkaways*, and is called *Tónkahuas* by Spanish writers. Through the unfortunate homophony of their name, they were frequently confounded with the Central Texan or coast tribe of the Towakonays, who certainly were congeners of the Wichitas and Wacoes, and led a nomadic life in close community with the Karankáhuas, Arrenámus and Caris. A bay in the middle part of the Texan Gulf Coast is called Carancáhua Bay up to this day. At the time when Spanish missionaries, along with a number of their Aztec helpmates, had colonized the South of Texas, and disseminated the germs of the Roman Catholic faith among the untutored tribes of aborigines, whom they induced to join in agricultural pursuits in the vicinity of their missions, the Indians were treated with humanity. Then Mexico and all Spanish America freed itself from the domination of the distant mother country; Texas declared itself independent from Mexico, and when after another lapse of time the Texan settlers proclaimed their adhesion to the American Union, a war of extermination commenced against the helpless Indians, which up to our days continues without abatement on the northern border against the roving bands of the Lipans, Comanches and Kiowas.

The Tonkawa tribe, however, whose first mention in American annals occurs at the commencement of this century, seems to have suffered more from internecine wars and feuds with the Comanches, than from white settlers. In 1847 official documents put down the number of their warriors at 155, a decrease of about two-thirds since 1820. The remnants of the tribe, about 35 warriors with their families, are with a number of Lipan-Apaches at present gathered on a reservation in Shackleford County, Northern Texas, seven miles from Fort Griffin. They raise stock, hunt the buffalo, and serve as scouts on the expeditions of the United States troops stationed at Fort Griffin. They are exceedingly filthy in dress and habits, paint their faces in a grotesque manner, and live in canvas tents. Their national legend represents them to be the offspring of the *wolf;* hence this animal is worshipped in their *wolf-dance,* of which Schoolcraft has given a description (in Vol. V).

Two Bavarian gentlemen have lately visited and studied this obscure and half-forgotten tribe, and have favored me with their notations on the na-